The Computational Modeling of Analogy-making

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Introduction

Our ability to see a particular object or situation in one context as being "the same as" another object or situation in another context is the essence of analogy-making. It encompasses our ability to explain new concepts in terms of already-familiar ones, to emphasize particular aspects of situations, to generalize, to characterize situations, to explain or describe new phenomena, to serve as a basis for how to act in unfamiliar surroundings, to understand many types of humor, etc. Within this framework, the importance of analogy-making in modeling cognition becomes clear.

Principles underlying the computational modeling of analogy

Analogy-making, whether human or computational, is typically conceived of as involving a *mapping* between two domains, called the *source* (or *base*) and the *target*. Hall (1989) lists four abstract processes that are widely considered to be necessary for analogical reasoning — namely, *recognition* of a source, given a target description; *elaboration* of the mapping between the two; *evaluation* of the mapping and *transfer* of information from the source to the target; and *consolidation* (i.e., learning) of the outcome. Chalmers, French, & Hofstadter (1992) suggested that this basic framework should also include *dynamic* representation-building mechanisms and *parallel* sub-process interaction.

The following (true) example anecdotally illustrates not only these processes, but also the ubiquity of analogical processing, even in completely ordinary situations: In 1973 I was, for the first time ever, in a European bathroom [target]. This obviously brought to mind [recognition] an American bathroom [source] because [elaboration and evaluation] the European bathroom sink clearly mapped to an American bathroom sink, the European bathtub (although having a somewhat different shape) mapped to an American bathtub, the European towel-rack to an American towel-rack, the European mirror to an American mirror, etc. However, one object in the European bathroom puzzled me. It was made of porcelain, had a drain, and could be rinsed out with water from two faucets. I concluded [transfer] that this object must be a European "toilet" and acted on my conclusion... (I only later discovered what a "bidet" was and realized that European toilets are frequently not in the bathroom.)

Classes of computational models of analogy-making

Although there are many ways of classifying analogy-making programs, we have chosen to classify them into three broad groups based on their underlying architectures. (For another classification scheme, see, for example, Spanoudakis & Constantopoulos, 1996) These are:

- "symbolic" models, so called because they are largely part of the "symbolic" paradigm in AI, in which symbols, logic, planning, search, means-ends analysis, etc.

play a predominant role. (See Hall, 1989, for an extensive review of these early models.)
- "connectionist" models that adopt, broadly speaking, the framework of the connectionist networks, including nodes, weights, spreading activation, etc.
- hybrid models that lie somewhere in between connectionist and symbolic models.

Symbolic models

The distinction of being the first computer model of analogy-making arguably goes to Argus (Reitman, 1965). The program solved proportional analogies that, by today's standards, seem trivial. For example, the program was given: bear:pig::chair:? and had do pick an answer from one of four choices: foot, table, coffee, strawberry. While the program was simple in the extreme, its architecture nonetheless included many far-sighted principles, among them the use of a conceptual network, interactions between the concept network and the problem to be solved, the realization of the necessity of automatic representation-building for the source and target, etc.

The best known model from the 1960's was ANALOGY (Evans, 1968). Like Argus, this program was designed to do proportional analogies of the form A:B::C:? taken from standardized high-school IQ tests (see Box 1). All of the objects in the analogies were simple geometric figures. One important feature of ANALOGY was that its input was a low-level description of each figure and, based on this, the program built a high-level description of the figure. All of the problems solved by ANALOGY are from the same domain, i.e., both source and target consist of geometric figures.

Also about that time, JCM (Becker, 1969) attempted to put the computational modeling of analogy-making into a more cognitively plausible, real-world framework, incorporating incipient notions of learning, Working Memory (WM) and a Long-term Memory (LTM) in which were stored representations of a set of primitive objects, importance-ranked relations between them, events and causal mappings.

A number of models from this early period drew heavily on formal logic. For example, ZORBA-1 (Kling, 1971) was an automated theorem prover that solved (target) problems by finding an analogous (source) problem, taking its proof and applying it to the target problem. Munyer (1981) and Greiner (1985) also developed analogy-making systems based on formal logic. Munyer's system, in particular, combined planning, problem-solving and deductive logic and implemented a process of gradual "convergence" to a correct mapping via an interaction between top-down (logic) and bottom-up, competitive processes.

The first attempt to apply production systems to computational analogy-making was ANA (McDermott, 1979) a program that did problem-solving in a micro-world. This program had an LTM knowledge base (stored as production rules) and a working memory. ANA progressively built the appropriate productions needed to solve the target task, analogous to a source task stored in LTM that it already knows how to do. It learned by saving the new productions in LTM.

Carbonell (1983) applied means-ends analysis to analogical retrieval. One key difference with previous work was that his transformational analogy method used weak search methods and sub-goaling to find solution *paths* to a solution to a particular target problem. The program had a stored set of second-order representations of solution-paths for previously solved problems. Means-ends analysis was then used to discover the source problem that best corresponded to the target problem. This transformational analogy method was later extended to a more powerful derivational analogy method (Carbonell, 1986; Veloso & Carbonell, 1990) that operated on automatically derived representations and included a peripheral knowledge base to improve the evaluation of various parts of the solution path.

MEDIATOR (Simpson, 1985) was the earliest application of case-based reasoning (CBR) to analogy-making. Prodigy/Analogy (Veloso & Carbonell, 1993) combined CBR and Carbonell's derivational approach to analogy-making. This architecture was explicitly designed to scale up to larger domains and a number of empirical results have shown that, as the number of stored episodes, the integrated learning system apparently did keep search requirements under control.

Winston (1978) developed the notion of *transfer frames*. Two objects (one source, one target) were presented to the program as being similes. Mappings were then made by the program between the source and target based on the most salient properties of the source, the prototypicality of the information in the target, and the instructional context provided by a tutor. After checking for inconsistencies with respect to these criteria, properties were transferred from the source to the target. This work was extended (Winston, 1980, 1982) to a model of analogical reasoning in which, in order to respond to a particular query, a rule was extracted from the source situation based on attributional and relational information in the source situation. This rule, based on consistent relational structure, was used to answer the target query.

Winston's work anticipated, in some sense, Gentner's (1983) Structure Mapping Theory (SMT). (See Box 2.) SMT is unquestionably the most influential work to date on the modeling of analogy-making and has been applied in a wide range of contexts ranging from child development to folk physics. SMT explicitly shifts the emphasis in analogy-making to the *structural* similarity between the source and target domains. Two major principles underlie SMT:

- the relation-matching principle: good analogies are determined by mappings of *relations* and not attributes (originally only identical predicates were mapped) and
- the systematicity principle: mappings of *coherent systems* of relations are preferred over mappings of individual relations.

This structural approach was intended to produce a domain-independent mapping process.

The Structure Mapping Engine (SME) was the computational implementation of SMT (Falkenhainer, Forbus & Gentner, 1989). More recent versions of SME have explored the use of pragmatics, as well as re-representation techniques that allow related, but not identical, predicates to match (Falkenhainer, 1990). MAC/FAC (Gentner & Forbus, 1991, Forbus, Gentner, & Law, 1995), a two-stage analogical retrieval engine, was later developed as a front-end to SME. The first stage of its retrieval process consists of a sweep through LTM retrieving many potential source episodes based on superficial search; the second stage is a detailed best-match process designed to select the best matches to the target. Only then does the structure mapping phase begin. MAGI (Ferguson 1994), another SME-based model, detects regularity within a given situation or scene by seeking maximally consistent mappings among its parts. Depending on the nature of the mappings found, elements of the scene can be categorized as being repetitions, or symmetrical. This structural notion of regularity applies to conceptual as well as perceptual materials.

IAM (Keane & Brayshaw, 1988; Keane et al, 1994) incrementally maps portions of a base domain to the source, thereby gradually building up a single interpretation based on selected portions of the domain rather than on many alternative interpretations. If the mapping produced is not optimal then this mapping will be abandoned and another constructed. The completely serial nature of IAM processing, however, has produced doubts about its ability to scale up (Forbus et al, 1994).

I-SME (Forbus et al, 1994) is an incremental version of SME based, in part, on the IAM architecture. The most significant difference with the latter program is that, instead of the strictly serial approach adopted by IAM, I-SME mixes serial and parallel processing.

In recent work another SME-based program, SEQL (Kuehne et al, 2000), has been applied to infant categorization. The authors suggest that categories are represented via structured descriptions and formed by a process of progressive abstraction, through successive comparison with incoming exemplars.

CARL (Burstein, 1986) extends the ideas of Gentner in a multi-stage analogy-making program that constructs analogies based on several analogies presented by a teacher in a somewhat context-dependent manner. Kedar-Cabelli's (1985) model of purpose-driven analogy attempts to automatically derive relevant structural and functional features in order to make mappings.

Recently, a "path-mapping" model (Salvucci & Anderson, 2001) of how humans integrate analogical mapping and problem solving has been developed based on ACT-R (Anderson, 1993). ACT-R has also been used in the context of analogy-making to attempt to develop a unified theory of metaphor understanding, semantic illusions and text memory (Budiu & Anderson, 2000) and to model invention by analogy (Murdock et al, 1998).

A final pair of symbolic models, BORIS (Lehnert et al, 1983) and MORRIS (Dyer, 1983), deserve mention. These programs attempt to understand narrative through the use of abstract "thematic abstraction units," which closely resemble Schank's (1982) Thematic Organization Points (TOPS) implemented in a dynamically organized memory. Analogies in these models are recognized largely through structural relations, rather than with simple attribute information.

Connectionist Models

Symbolic systems are generally well equipped to model relational structures involving situations represented as objects and relations between objects. For this reason, these models held the high ground for many years in the computational modeling of analogy-making. However, due largely to recent advances in their representation techniques, connectionist models have taken their place alongside symbolic models of analogy-making. Most importantly, distributed connectionist representations provide a natural internal measure of similarity, thereby allowing the system to handle with relative ease the problem of similar, but not identical, relations, a problem that has proved difficult for symbolic models.

ACME (Holyoak & Thagard, 1989) was the first attempt to develop an architecture in which analogy-making was an emergent result of constrained, parallel activation of states of in a neural network-like structure. In this model, structural similarity, semantic similarity, and pragmatic importance determine a set of constraints to be simultaneously satisfied. The model is supplied with representations of the target and source and proceeds to build a localist constraint-satisfaction network in which hypothesis nodes correspond to all possible hypotheses pairing the elements of the source with those of the target. Excitatory and inhibitory links between these nodes implement the constraints. In this way, contradictory hypothesis nodes compete and do not become simultaneously active, while consistent nodes mutually support each other. The relaxation of the network provides a parallel evaluation of all possible mappings and finds the best one, represented by the set of most active hypothesis nodes. ARCS (Thagard et al, 1990) is a model of retrieval that is coupled with ACME in which mapping is dominated by structural similarity and retrieval is dominated by semantic similarity.

One of the most ambitious connectionist models of analogy-making, LISA (Hummel & Holyoak, 1997), can reasonably be called a descendant of ACME. Whereas ACME required all objects in the source to be pairwise connected to all elements in the target, LISA relies on more plausible mechanisms, such as partially distributed representations of concepts, selective activation and dynamic binding as the means of associating the relevant structures.

Only node structures that *oscillate in synchrony* are bound together (Shastri & Ajjanagadde, 1993; Sougné, 1996). Crucially, the synchronous binding mechanism means that both WM and LTM can interact during both retrieval and mapping. LISA successfully integrates the process of retrieval of a base and the mapping of the base and target.

STAR-1, designed to solve proportional analogies, was the first distributed connectionist model of analogy-making (Halford, et al, 1994) and is based on the tensor product connectionist models developed by Smolensky (1990). STAR-2 (Wilson, et al., 2001) is a recent and more complex version of STAR-1, developed in an attempt to achieve a better understanding of the development of analogy-making capabilities in children.

DRAMA (Eliasmith & Thagard, 2001) is a recent connectionist model of analogy-making that implements holographic reduced representations (Plate, 1995), a type of convolution-correlation memory (Metcalfe-Eich, 1985). This program, using fully distributed representations of concepts, attempts to integrate the semantics and structure of the base and target during the mapping process.

Jani & Levine (2000) have developed a neural network approach to analogy-making based on Adaptive Resonance Theory (Carpenter & Grossberg, 1986). This system has a concept association mechanism based on synaptic triads, and explicitly appeals to neurobiological plausibility. Analogator (Blank, 1996) is a connectionist model that learns to make analogies by seeing numerous analogies.

Hybrid Models

Hybrid models share features of both connectionist and symbolic models. (The term "connectionist" here is meant to be broadly construed, encompassing architectures that rely on connectionist-like mechanisms such as spreading activation among node structures, excitation and inhibition between nodes, etc.) The first two models discussed here rely on agent-based approaches to analogy-making.

COPYCAT (Mitchell, 1993; see Box 3), TABLETOP (French, 1995), LETTER-SPIRIT (McGraw, 1995), and METACAT (Marshall & Hofstadter, 1997) form a family of models whose basic architectural principles were described by Hofstadter (1984, 1995). Three of the most important features of these models of analogy-making are i) their ability to build up their own representations of the source and target as well as the mapping between them via an agent-driven interaction between top-down (LTM) and bottom-up (WM) processing, ii) their use of (simulated) parallelism, and iii) their stochastic nature. These models abandon traditional sequential processing and allow representation-building and mapping to run in parallel and to continually influence each other. In this way, partial mappings can have an impact on further representation-building (and vice-versa), thus allowing the gradual construction of context-sensitive representations.

AMBR (Kokinov, 1988), an analogical problem-solver, is based on the principles of the DUAL model (Kokinov, 1994), a general, context-sensitive cognitive architecture consisting of many micro-agents each of which represents a small piece of knowledge. Each micro-agent has a symbolic part that encodes the declarative and/or procedural knowledge it is representing and a connectionist part that computes the agent's activation level, which represents the relevance of this knowledge to the current context. The AMBR model, and its later extension, AMBR-2 (Kokinov & Petrov, 2000), implements the interactive parallel processes of recollection, mapping and transfer that emerge from the collective behavior of the agents and the result of which is an analogy, but also a re-representation of the old episode which may turn out to be illusory memory.

Other hybrid models that combine symbolic and connectionist mechanisms, use spreading activation mechanisms, node structures implementing knowledge bases, etc., include ASTRA (Eskeridge, 1994) and ABR-Conposit (Barnden, 1994). ASTRA implements "continuous analogical reasoning" that recognizes the importance of integrating the various stages of analogy-making rather than treating them independently. ABR-Conposit (Barnden, 1994) is an implementation of Analogy-Based Reasoning that implements WM-WM matching, creates and modifies WM representations, and manipulates complex data structures in an explicit attempt to bridge the symbolic-connectionist gap.

Conclusion

We have presented a brief, and necessarily incomplete, survey of computational models of analogy-making over the last 35 years. These models are divided into three broad classes: those who architectures are based largely on the principles of the symbolic tradition of artificial intelligence, those that draw on connectionist principles and hybrid models that depend on a combination of these principles. Great challenges lie ahead for the field, among them, the development of context-sensitive ways for analogy programs to converge on precisely the "right" representations that allow a particular analogy to be made, the systematic incorporation of learning mechanisms into the programs, and, of course, the development of programs that can effectively scale up.

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References

Anderson, J. R. (1993). *Rules of the Mind*. Hillsdale, NJ:Lawrence Erlbaum.

Barnden, J. A. (1994). On the connectionist implementation of analogy and working memory matching. In J. A. Barnden, & K. J. Holyoak (Eds.) *Advances in Connectionist and Neural Computation Theory Volume 3: Analogy, metaphor, and reminding.* Norwood, NJ: Ablex Publishing Corporation.

Becker, J. D. (1969). The modeling of simple analogic and inductive processes in a semantic memory system. *Proceedings of IJCAI-69*, Washington, DC, 655-668.

Blank, D.S. (1997). Learning to see analogies: a connectionist exploration. Unpublished doctoral dissertation, Indiana University, Bloomington, IN.

Budiu, R. & Anderson, J. R. (2000). Integration of background knowledge in sentence processing: A unified theory of metaphor understanding, semantic illusions and text memory. In *Proceedings of the Third International Conference on Cognitive Modeling*, University of Groningen, Netherlands, 50-57.

Burstein, M. (1986). Concept formation by incremental analogical reasoning and debugging. In *Machine Learning: An artificial intelligence approach.* R. Michalski, J. Carbonell, T. Mitchell (eds.), vol. 2, ch. 13, 351-70. Los Altos, CA: Morgan Kaufman.

Carbonell J.G., *Derivational Analogy: A Theory of Reconstructive Problem Solving and Expertise Acquisitio* n, Machine Learning: An Artificial Intelligence Approach, Vol. II, Morgan Kaufmann Publishers Inc., Los Altos, California, 1986

Carbonell J.G., *Learning by Analogy: Formulating and Generalizing Plans from Past Experience*, Machine Learning: An Artificial Intelligence Approach, Vol. I, Morgan Kaufmann Publishers Inc., Los Altos, California, 1983

Carpenter G. and Grossberg, S. (1986). Adaptive resonance theory: Stable self-organization of neural recognition codes in response to arbitrary list of input patterns. In *Proceedings of the Eighth Annual Conference of the Cognitive Science Society*. Hillsdale, NJ: LEA. 45-62.

Chalmers, D. J., French, R. M. and Hofstadter, D. R. (1992). High-level Perception, Representation, and Analogy: A Critique of Artificial Intelligence Methodology. *Journal of Experimental and Theoretical and Artificial Intelligence*, *4*(3), 185-211.

Dyer, M. (1983). Understanding stories through morals and remindings. *Proceedings IJCA-83*, 75-77.

Eliasmith, C. & Thagard, P. (2001). Integrating structure and meaning: a distributed model of analogical mapping. *Cognitive Science, 25*, 245-286.

Eskeridge, T. (1994). A hybrid model of continuous analogical reasoning. In Holyoak, K. and Barnden. J. (Eds.) *Advances in Connectionist and Neural Computation Theory*. *Volume 2: Analogical Connections*, NJ:Ablex, 207-246.

Evans, T. (1968). A program for the solution of a class of geometric analogy intelligence test questions. In *Semantic Information Processing*. 271-353. Cambridge, MA: MIT Press.

Falkenhainer, B. (1990) Analogical interpretation in context. *Proceedings of the Twelfth Annual Conference of the Cognitive Science Society*, NJ: LEA, 69-76.

Falkenhainer, B., Forbus, K. D. & Gentner, D. (1989). The structure-mapping engine. *Artificial Intelligence, 41* (1), 1-63.

Ferguson, R. W. (1994). MAGI: Anology-based encoding using symmetry and regularity. *Proceedings of the Sixteenth Annual Conference of the Cognitive Science Society*, NJ:LEA, 283-288.

Forbus, K. & Ferguson, R. & Gentner, D. (1994). Incremental Structure-Mapping. Proceedings of the Sixteenth Annual Meeting of the Cognitive Science Society. NJ: LEA, 313-318.

French, R. (1995). *The Subtlety of Sameness: A theory and computer model of analogy-making*. Cambridge, MA: MIT Press.

Gentner, D. & Forbus, K., (1991). MAC/FAC: A model of similarity-based access and mapping. *Proceedings of the Thirteenth Annual Cognitive Science Conference,* 504-509, Hillsdale, NJ: Lawrence Erlbaum Associates.

Gentner, D. (1983). Structure-mapping: A theoretical framework for analogy. *Cognitive Science, 7* (2). 155-70.

Greiner R., *Learning and Understanding by Analogies*, Artificial Intelligence, 35, 1988

Halford, G., Wilson, W., Guo, J., Gayler, R., Wiles, J., Stewart, J. (1994). Connectionist implications for processing capacity limitations in analogies. In: K. Holyoak & J. Barnden (eds.) *Advances in connectionist and neural computation theory, vol. 2, Analogical Connections,* pp. 363--415. Norwood, NJ: Ablex.

Hall, R. (1989). Computational approaches to analogical reasoning: a comparative analysis. *Artificial Intelligence, 39*, 39-120.

Hofstadter D. & the Fluid Analogies Research Group. (1995). *Fluid Concepts and Creative Analogies: Computer Models of the Fundamental Mechanisms of Thought*. Basic Books, New York.

Hofstadter, D. R. (1984). *The Copycat project: An experiment in nondeterminism and creative analogies.* MIT AI Memo No. 755, Cambridge, MA.

Holyoak, K. J., & Thagard, P. (1989). Analogical mapping by constraint satisfaction. Cognitive Science, 13, 295-355.

Hummel, J. E. & Holyoak, K.J. Distributed representations of structure: A theory of analogical access and mapping. Psychological Review, 104, 427-466, 1997

Jani, N. & Levine, D. (2000). A Neural Network Theory of Proportional Analogy-Making. Neural Networks, 13, 149-183.

Keane, M. & Brayshaw, M.. (1988). The Incremental Analogical Machine: A computational model of analogy. In D. Sleeman (Ed.), European Working Session on Learning. Pitman, 1988.

Keane, M. T., Ledgeway, T., and Duff, S. (1994). "Constraints on Analogical Mapping: A comparison of Three Models", Cognitive Science, 18, 387-438, 1994.

Keane, M., Ledgeway, T. & Duff, S.. (1994). Constraints on analogical mapping: A comparison of three models. Cognitive Science, 18, 387-438.

Kedar-Cabelli, S. (1985). Purpose-directed analogy. In *Proceedings of the Seventh Annual Conference of the Cognitive Science Society. Hillsdale, NJ: LEA*, 150-159.

Kenneth D. Forbus, K., Gentner, D., Law, K. (1995). MAC/FAC: A Model of Similarity-Based Retrieval. Cognitive Science 19(2): 141-205.

Kling, R. (1971). A paradigm for reasoning by analogy. *Artificial Intelligence*, *2*, 147-178.

Kokinov, B. & Petrov, A. (2000). Dynamic extension of episode representation in analogy-making in AMBR. *Proceedings of the 22nd Annual Conference of the Cognitive Science Society*, 274-279. NJ: LEA.

Kokinov, B. (1988). Associative memory-based reasoning: How to represent and retrieve cases. In T. O'Shea & V. Sgurev (eds.) *Artificial Intelligence III: Methodology, systems, and applications*. 51-58. Amsterdam: Elsevier.

Kokinov, B. (1994). The context-sensitive cognitive architecture DUAL. In Proceedings of the Sixteenth Annual Conference of the Cognitive Science Society. NJ:LEA, 502-507.

Kuehne, S. E., Forbus, K. D., Gentner, D., & Quinn, B. (2000, August). SEQL: Category learning as progressive abstraction using structure mapping. *Poster session presented at the Twenty-Second Annual Conference of the Cognitive Science Society,* Philadelphia, PA. Lawrence Erlbaum Associates

Lehnert, W.,Dyer, M., Johnson, P., Yang, C. & Harley, S. (1983). BORIS --- An Experiment in In-Depth Understanding of Narratives. Artificial Intelligence, 20, pp. 15-62.

Marshall, J. and Hofstadter, D. (1997). The Metacat Project: A Self-Watching Model of Analogy Making. Cognitive Studies: Bulletin of the Japanese Cognitive Science Society, special issue on similarity and analogical reasoning, 4(4), 57-71

McDermott J., Learning to Use Analogies, Proceedings of the International Joint Conference on Artificial Intelligence(IJCAI-79), Tokyo, 1979

McGraw, G. (1995). Letter Spirit (part one): Emergent High-Level Perception of Letters Using Fluid Concepts. Ph.D. dissertation, Indiana University, Bloomington, IN. http://www.cogsci.indiana.edu/farg/mcgrawg/thesis.html.

Metcalfe Eich, J. (1985) Levels of processing, encoding specificity, elaboration and CHARM. Psychological Review, 92(1): 1-38.

Mitchell, M. (1993) Analogy-making as Perception: A computer model. Cambridge, MA: MIT Press.

Munyer, J. (1981). Analogy as a means of discovery in problem-solving and learning. Ph.D dissertation. UC Santa Cruz, CA.

Murdock, J. W., Simina, M., Davies, J., & Shippey, G. (1998). Modeling Invention by Analogy in ACT-R. In Proceedings of the Twentieth Annual Conference of the Cognitive Science Society. Mawhaw, NJ:LEA. 740-745.

Plate, T. (1995). Holographic Reduced Representations. IEEE Transactions on Neural Networks, 6(3), 623-641

Reitman, W. R. (1965). *Cognition and thought: an information processing approach.* New York, NY: John Wiley and Sons.

Salvucci, D. & Anderson, J. (2001). Integrating analogical mapping and general problem-solving: the path-mapping theory. *Cognitive Science, 25*(1), 67-110.

Schank, R. (1982). Dynamic memory: a theory of reminding and learning in computers and people. UK: Cambridge University Press.

Shastri, L. & Ajjanagadde, V. (1993). From Simple Associations to Systematic Reasoning: A connectionist representation of rules, variables and dynamic bindings using temporal synchrony. *Behavioral and Brain Sciences,16,* 417-494.

Simpson, R. L. (1985). A Computer Model of Case-Based Reasoning in Problem Solving: An Investigation in the Domain of Dispute Mediation. Technical Report GIT-ICS-85/18, Georgia Institute of Technology, School of Information and Computer Science, Atlanta, USA.

Sougné, J. (1996). A connectionist model of reflective reasoning using temporal properties of node firing. *Proceedings of the Eighteen conference of the Cognitive Science Society*. NJ:LEA, 666-671 .

Smolensky, P. (1990). Tensor product variable binding and the representation of symbolic structures in connectionist systems. *Artificial Intelligence, 46* (1-2), 159-216.

Spanoudakis G., Constantopoulos P.: Elaborating Analogies from Conceptual Models, International Journal of Intelligent Systems, Vol. 11, No 11, pp. 917-974, (ed) Yager R., J.Wiley and Sons, 1996

Thagard, P., Holyoak, K., Nelson, G., & Gochfeld, D. (1990). Analog retrieval by constraint satisfaction. Artificial Intelligence, 46, 259-310.

Veloso, M. and Carbonell, J., 1990. Integrating analogy into a general problem-solving architecture. In Maria Zemankova and Zbigniew Ras, editors, Intelligent Systems, UK: Ellis Horwood, Chichester. 29-51.

Veloso, M.M. & Carbonell, J.G.(1993). Derivational Analogy in PRODIGY: Automating Case Acquisition, Storage, and Utilization. Machine Learning, 10(3):249-278.

Wilson, W., Halford, G. Gray, B., Phillips, S. (2001). The STAR-2 Model for Mapping Hierarchically Structured Analogs. In: D. Gentner, K. Holyoak, B. Kokinov (eds.) *The Analogical Mind.* Cambridge, MA: MIT Press.

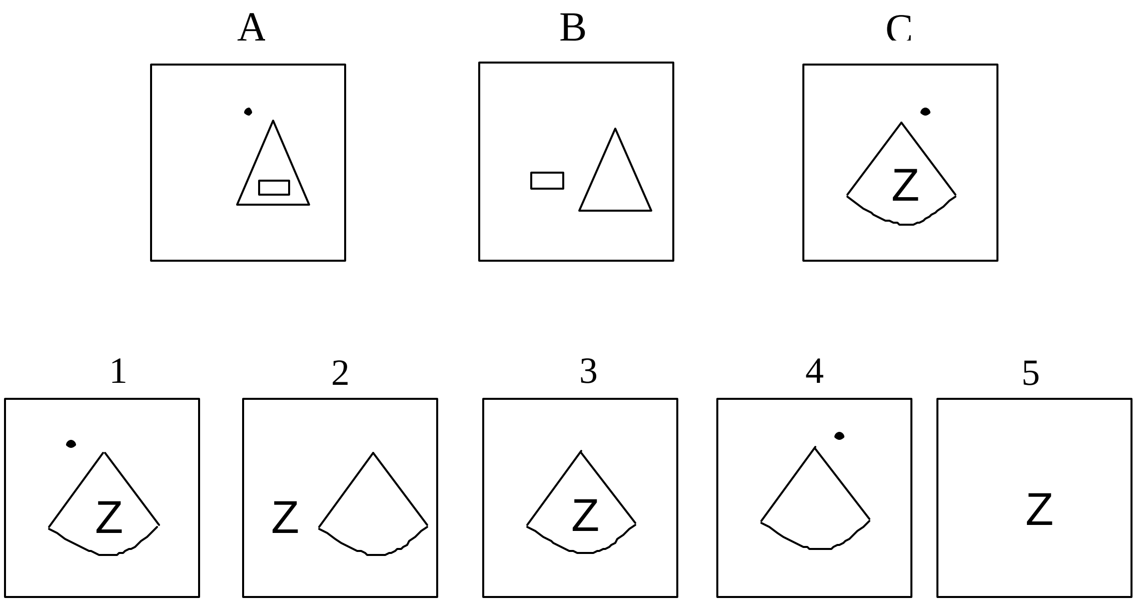
Winston, P. H. (1978). Learning by creating and justifying transfer frames, Artificial Intelligence, 10(2), 147-172.

Winston, P. H. (1980). Learning and reasoning by analogy, *Communications of the ACM, 23*(12), 689-703.

Winston, P. H. (1982). Learning new principles from precedents and exercises, *Artificial Intelligence, 19*, 321-350.

Box 1

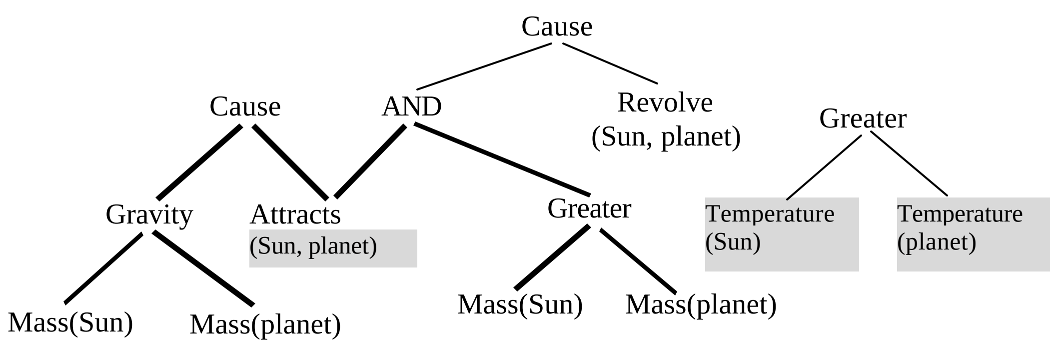
ANALOGY (Evans, 1968), one of the earliest analogy-making programs attempts to "construct a rule which transforms Figure A into Figure B and Figure C into exactly one of the five answer figures." The representation module first analyzes the input (written as a low-level description, rather than being the actual figures) and describes figure A, for example, as ((inside P2 P3) (above P1 P3) (above P1 P2)). Similar representations are made for figures B and C and for the five test figures. Based on these representations, the program matches the most similar descriptions in order to discover the correct analogy. Notice that the program has no semantic knowledge about the figures it manipulates. For example, it does not know that squares and rectangles are generally closer in people's minds than, say, squares and letters.



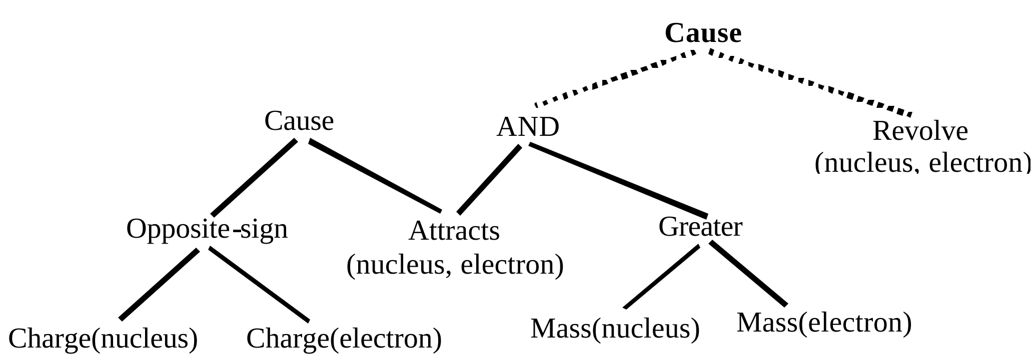
Box 2

SME, the Structure Mapping Engine (Falkenheimer et al, 1989), a computational implementation of the Structure Mapping Theory (Gentner, 1983) has been the most influential computational model of analogy-making to date. It receives predicate-calculus representations of the base and source and searches both representations to determine where there are structural similarities between them. It builds a mapping between the two situations based on these structures and their overall coherence. Discovering two matching systematic structures (heavy lines) in the source (Solar system) and in the target (Rutherford atom) allows the program to transfer structure found in the source to the target (in this case, to conclude that the cause of the electron revolving around the nucleus is the charge). It is hard to know how what conclusions SME might have drawn if the representation of the Rutherford atom had also included the fact that, in addition to electrical forces, there are gravitational forces between the nucleus and the electron (for a discussion of this point, see French, 1995).

SOLAR SYSTEM



RUTHERFORD ATOM



Box 3

COPYCAT (Hofstadter, 1984; Mitchell, 1993) solves letter-string analogies of the form: ABC:ABD::KJI:? and gives probabilistically possible answers like LJK, KJJ, KJD, etc. The architecture of COPYCAT involves a working memory, a semantic network (simulating LTM) defining the concepts used in the system and their relationships, a procedural memory storing small, nondeterministic computational agents ("codelets") that build, examine and, possibly, destroy the structures in the working memory and continually interact with the semantic network. The system gradually settles towards a set of consistent set of structures that will determine the mapping between the base and the target.